Signal Sampling and Aliasing

Problem

In this digital age, vast amounts of data are generated and processed every day, and signal processing is crucial in analyzing and understanding this data. Whether it's analyzing audio signals, images, or other types of data, signal processing techniques such as sampling and reconstruction, harmonic analysis, and interpolation play a vital role in accurately representing and analyzing the underlying information. Understanding these techniques and their limitations is essential in ensuring that the data we process and analyze is represented accurately and meaningfully.

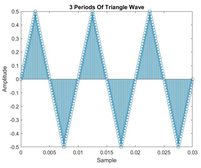
Introduction

In this study we examine signal harmonic analysis in time invariant signals as well as time variant and aliased signals. Aliasing in visual signals is also observed and multiple interpolation methods are compared visually to determine the values of each method.

Analyzing harmonics in periodic signals

Method

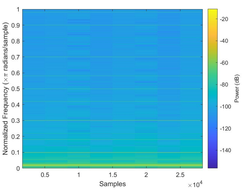
A triangle wave is generated with a period of 10ms and sampled at a rate of 1KHz for 3 seconds. The fundamental frequency of this signal can be determined by examining the power transmitted at the various component frequencies. The greatest power will be transmitted at the fundamental frequency.

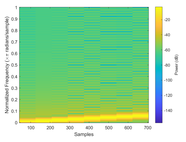


To perform this analysis, the DFT is taken of the sampled signal and plotted over a spectogram. The yellow line indicates the strongest signal. This same analysis is also performed for an FM chirp signal of varying frequencies.

Results

It can be seen from the spectogram on the right that the greatest power corresponds to a frequency of 100 Hz which is the natural frequency of the signal. The same analysis can be done on time variant signals as shown below.



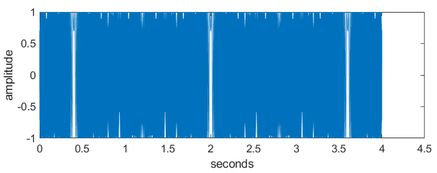


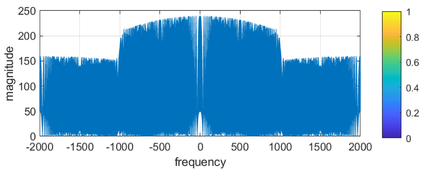
From the spectogram on the left, the power intensity can be seen to shift with the change in frequency.

Analyzing harmonics in time variant and aliased signals

Method

An FM Chirp signal is generated with a frequency ranging from 1KHz to 11KHz over a span of 4 seconds. The same signal is sampled at a constant 4KHz sampling rate for 4 seconds. The linear and log magnitude plots of the sampled signal are compared and the effects of folding and aliasing are observed.





Results

Due to the effects of signal folding and aliasing, the frequency analysis does not reveal the total frequency range up to 11KHz. This highlights the importance of selecting the correct sampling frequencies for a given signal.

Signal aliasing in visual signals

Method

An image is loaded into Matlab and downsampled by taking every third pixel in every row and column of the image. After resampling, aliasing appears visually as distortions or artifacts that were not present in the original image.

An approximation of the original signal is derived via a zero hold, or nearest neighbor, interpolation. The zero hold interpolation is compared to a linear interpolation visually.

Results

Visually, the reconstructed image appears to be smoother than the downsampled image. However, when compared to the original image, we can observe some blurriness and loss of sharpness in the reconstructed image. Some of the details in the original image may also be lost in the reconstruction process. The reconstruction process does not remove the aliasing effects completely.

In general, the linear interpolation produces a smoother and more visually appealing reconstruction compared to the zero-order hold interpolation.

Conclusion

In conclusion, the importance of signal processing cannot be overstated in our digital world. The techniques of signal sampling, harmonic analysis, and interpolation play critical roles in accurately representing and analyzing data in various forms. Understanding the limitations of these techniques is just as important as their applications. The study of signal harmonic analysis in periodic signals, as well as time-variant and aliased signals, sheds light on the complexity of signal processing and the challenges that arise in practice. The comparison of multiple interpolation methods to mitigate the effects of aliasing in visual signals highlights the practical importance of these techniques. By continuing to research and refine signal processing methods, we can ensure that we accurately represent and analyze the vast amounts of digital data that we generate every day.

Contributions

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